1. Amoco Corporation, the fifth largest oil company in the US, whose goal is to streamline its assets during a period of low prices by divesting properties optimized for smaller, low-cost companies while targeting more productive assets. On the other hand, Apache, known for its efficiency in managing small- to medium-sized properties, pursued an acquisition strategy to enhance operational efficiency. The acquisition of MW properties by Apache aligned with its "rationalize and configure" approach, promising cost savings, improved cash flows, and reduced volatility. With increased reserves and cost-efficient strategies, Apache stood to benefit from real options and exploration opportunities, enhancing production rates and flexibility. Amoco, recognizing the inefficiency of managing MW petroleum, saw value in selling to Apache to increase cash flows. Both parties mutually beneficial impact to cash flows.

2 *Estimate a DCF valuation for each of the four categories of MW Petroleum reserves — proved developed, proved undeveloped, probable, and possible (Exhibits 3-6). You should use the projections given in Exhibits 3-6 for your valuation. You can exclude the tax shields from debt financing and the "additional exploration opportunities" from your valuation. Based on the valuations you compute for the four categories, how much should Apache be willing to pay in the acquisition? Do you think this estimate is more likely to be biased high or low?*

Based on the provided forecasts and a DCF analysis, estimate that Apache should be willing to pay at most $**$743.67M** (see Figure 2.1 - 2.4) for this acquisition. This DCF Is likely biased too high, given that the forecasts for all four categories are incredibly volatile and have different likelihoods of success. Further, the only proven developed category is showing declining cash flows in all years, starting from $80M and declining to a forecasted cash flow of $22M, before any discounts. All other categories are either underdeveloped or unproven, and carry a high risk of not panning out for Apache, which must be taken into consideration into the valuation. In addition, the discount rates used are high given the high volatility of the market, future oil prices, and uncertain long-term growth prospects in general that impact this valuation. Even with these considerations in the discount and future growth rates, the likelihood of bias driven by the upfront Cash Flow forecasts impact the valuation considerably.

3. When valuing MW as a portfolio composed of assets-in-place and real options, we need to differentiate between these components based on their attributes and the capital expenditure needed. Assets-in-place primarily include *Proved Development Reserves* which are operational and revenue-generating with minimal additional investment needed. These assets are comparable to machinery that is fully paid-for which is already producing output.

Real options, on the other hand, require significant exercise price to realize potential future cash flows. These include *Proved Undeveloped Reserves* which are proven but require additional development, in addition to *Probable and Possible Reserves* which carry higher uncertainty and cost, but offer more flexibility in terms of when development is needed. There are also *Exploration Opportunities* which represent real options, enabling the company to explore and possibly extend fields; these are highly dependant on the overall market dynamics (current and projected), as well as technological improvements.

Real Options Valuation typically yields a *higher valuation* than DCF and this is because it factors in managerial flexibility to adjust its investment strategies based on market response and technological changes - these are factors that DEC doesn't cover, as it focuses more on estimating the present value of expected future cash flows under a stationary strategy. The Adjusted Present Value method overlooks the strategic advantages of ROV although it does adjust the NPV for financing effects; it also does not capture the value of deferring investments, scaling operations up or down, and other strategic considerations.

The source of the option value in ROV derives from the volatility in underlying resource prices and potential technological advancements. High volatility increases the value of holding an option, because the chance of favorable price movements that make high-cost investments more profitable increases. In this way, technological progress can reduce operational costs and enable previously non-viable options, furthering the option value.

4.
For our Black-Scholes model we used the following inputs

Formula Input	**Derivation**
S	The Present Value of the Cashflows
K	The First 5 years of CapEx
rf	The weighted average of the 10 and 30 year treasury
T	1 year offering period
σ	The historical volatility of oil prices

b) We chose our volatility measure as the average of historical oil prices, prior to regional conflict which we believe is increasing volatility in the short term, but will decrease long term. The volatility should represent the volatility of the cash flows from the assets, however since these aren't directly observable we used the price of oil as a proxy. We believe that since overhead expenses only represent a small portion of the overall expense of these assets, the volatility of the cash flows would be highly correlated with oil's volatility.
c) As volatility (σ) and maturity (T) increase the value of the option increases. (Figure 4.1 Sensitivity Tables). In general, extending the maturity by 1 year increases the value of the option more than a 1 percentage point increase in volatility.
d) None of the resources has a higher real option value than the DCF valuation. (FIgure 4.3). This is because they are not able to defer the decision to invest in the project and T=1 in the black scholes equation.

e) Apache should be willing to pay between $1,077.18 and $1,548.63 million for the resources alone. This would be an appropriate range between the real option value and the DCF value for the expected cash flows. (Figure 4.3)

Appendix

Figure 2: DCF Assumptions

2.1 CAPM and Terminal Growth Rate Assumptions

Discount and Growth Rate Assumptions	**Value**	**Rationale**
Unlevered Asset Beta	0.82	Given
RF (Risk Free Rate)	8.19%	weighted average of all 10 and 30 yr govt bond yields
MRP	10%	Uses Feb '91 BBB Industrial Bond Yield as Proxy
Discount Rate	16.70%	CAPM
Terminal Growth Rate	2%	Assumed

2.2 Risk Free Rate Calculation

Risk Free Rate		
R	**T**	
8.03%	10	
8.24%	30	
8.19%		Weighted Average

2.3 DCF

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
	$91	**$92**	**$93**	**$94**	**$95**	**$96**	**$97**	**$98**	**$99**	**$0**	**$1**	**$2**	**$2**	**$4**	**$5**	**06 (Term)**
Proved Develop DCF																
Cash Flow	$89	$80	$70	$63	$56	$50	$45	$42	$38	$34	$34	$30	$26	$24	$22	$20
Discounted Cash Flow	$77	$59	$44	$34	$26	$20	$15	$12	$9	$7	$6	$5	$4	$3	$2	
Proved Undeveloped DCF																

Cash Flow	-$14	-$6	$8	$5	$5	$7	$10	$13	$16	$15	$7	$15	$15	$17	$15	$18
Discounted Cash	-$12	-$4	$5	$3	$2	$3	$3	$4	$4	$3	$1	$2	$2	$2	$1	
Probables																
Cash Flow	-$6	$4	-$2	-$3	$10	$12	$13	$13	$13	$11	$10	$10	$8	$7	$6	$6
Discounted Cash Flow	-$5	$3	-$1	-$1	$4	$5	$4	$4	$3	$2	$2	$2	$1	$1	$1	
Possibles DCF																
Cash Flow	-$9	-$3	-$14	-$28	-$17	$3	$13	$13	$19	$16	$21	$20	$19	$17	$14	$13
Disocunted Cash Flow	-$7	-$2	-$8	-$15	-$8	$1	$4	$4	$5	$3	$4	$3	$3	$2	$1	
Total	**$53**	**$55**	**$39**	**$20**	**$25**	**$28**	**$27**	**$23**	**$21**	**$16**	**$13**	**$12**	**$9**	**$7**	**$6**	

PV Cash Flows	$355.05	(Sum '91-05 Discounted Cash Flows)
Terminal Value Cash Flows	$57.12	(Sum of '06 Terminal Cash Flows)
PV Terminal Value	$388.61	(Terminal Value / (r-g))
Total Value (PV Cash Flows + PV Terminal Value)	**$743.67**	PV '91-05 CF + PV Terminal CF

Figure 2.4 Details on Terminal Value DCF

Terminal Value Estimation = '05 Value* (1+Last 5 Year CAGR)							
Given volatility and declining forecast in final years, use only periods 11-15							
	'01 ($M)	*'02*	*'03*	*'04*	*'05*	*'06 (Terminal)*	CAGR '01-'05
Proven Developed	$34.20	$29.50	$26.10	$23.50	$22.10	*$20.25*	-8.36%
Proved Undeveloped	$6.80	$14.70	$15.20	$16.50	$15.10	*$17.71*	17.30%
Probables	$10.20	$10.00	$8.40	$7.00	$6.40	*$5.83*	-8.90%
Possibles	$21.20	$20.10	$18.80	$16.80	$14.40	*$13.33*	-7.44%

Figure 4.1 Real Option Value Sensitivity Tables

Proved DevelopedResources Sensitivity

669.795	**1.00**	**2.00**	**3.00**	**4.00**	**5.00**
15%	$ 368.61	$ 368.61	$ 368.63	$ 368.91	$ 369.79
20%	$ 368.65	$ 368.66	$ 368.75	$ 369.42	$ 370.88

25%	$ 368.70	$ 368.70	$ 368.93	$ 370.06	$ 372.01
30%	$ 368.74	$ 368.75	$ 369.19	$ 370.77	$ 373.07
35%	$ 368.79	$ 368.81	$ 369.50	$ 371.49	$ 374.03
Proved Undeveloped Resources Sensitivity					
122.046	**1.00**	**2.00**	**3.00**	**4.00**	**5.00**
15%	$ 33.45	$ 39.34	$ 46.61	$ 53.63	$ 59.85
20%	$ 34.27	$ 41.66	$ 50.02	$ 57.64	$ 64.03
25%	$ 35.10	$ 43.75	$ 52.90	$ 60.87	$ 67.20
30%	$ 35.93	$ 45.63	$ 55.40	$ 63.52	$ 69.65
35%	$ 36.73	$ 47.35	$ 57.58	$ 65.74	$ 71.58
Probable Resources Sensitivity					
97.147	**1.00**	**2.00**	**3.00**	**4.00**	**5.00**
15%	$ 38.64	$ 43.02	$ 49.30	$ 55.61	$ 61.30
20%	$ 39.19	$ 44.99	$ 52.35	$ 59.27	$ 65.15
25%	$ 39.79	$ 46.80	$ 54.96	$ 62.24	$ 68.08
30%	$ 40.41	$ 48.45	$ 57.23	$ 64.68	$ 70.36
35%	$ 41.03	$ 49.98	$ 59.22	$ 66.73	$ 72.15
Possible Resources Sensitivity					
35.672	**1.00**	**2.00**	**3.00**	**4.00**	**5.00**
15%	$ 48.20	$ 63.70	$ 80.00	$ 95.05	$ 108.17
20%	$ 50.63	$ 69.01	$ 87.34	$ 103.52	$ 116.91
25%	$ 52.91	$ 73.65	$ 93.50	$ 110.30	$ 123.50
30%	$ 55.05	$ 77.79	$ 98.77	$ 115.84	$ 128.60
35%	$ 57.06	$ 81.53	$ 103.37	$ 120.46	$ 132.61

Figure 4.2 Black-Scholes Values of Resources

		Proved Developed	**Proved Undeveloped**	**Probable Reserves**	**Possible Reserves**
Current price	S	$680.13	$166.41	$136.16	$133.76
Initial CapEx (Exercise price)	K	$11.20	$48.10	$42.30	$108.20
Standard deviation	σ	0.25	0.25	0.25	0.25

(annual)					
Risk-free rate	r	8.08%	8.08%	8.08%	8.08%
Time to expiration (years)	T	1	1	1	1
	PV(K)	10.330	44.365	39.016	99.799
	d_1	16.874	5.413	5.125	1.297
	d_2	16.624	5.163	4.875	1.047
European Call price	**C**	**669.795**	**122.046**	**97.147**	**35.672**
European Put price	**P**	**0.000**	**0.000**	**0.000**	**1.708**
Put-call parity check for put		0.000	0.000	0.000	1.708

Figure 4.3 Total Value of Assets

Comparative Value of Reserves					
DCF Valuation	**$1,116.46**	$680.13	$166.41	$136.16	$133.76
Real Option Value	**$924.66**	$669.79	$122.05	$97.15	$35.67
Delta	**$191.80**	$10.33	$44.37	$39.02	$98.09